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LETTER TO THE EDITOR

Kinetic growth transition in a simple aggregation of charged particles with injection

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Abstract. The simple aggregation model of charged particles introduced by Takayasu is explored using computer simulations. In the Takayasu model, positive or negative charges are randomly injected at a constant fraction p (p: the fraction of positive charges) and each cluster having a positive or negative charge is conserved when two particles collide. We find a kinetic growth transition between cluster growth of positive charges and that of negative charges at the critical concentration $p_c = 0.5$. For $p > p_c$ (or $p < p_c$), the cluster size distribution of a positive charge (or a negative charge) shows the dynamic scaling $n_S(t) \approx S^{-\tau} f(S/t^z)$ with the same exponents ($\tau = \frac{4}{3}$ and $z = \frac{3}{2}$) as the Scheidegger river model at longer timescales than the correlation time t_c where $n_S(t)$ indicates the cluster distribution with a positive charge S (or a negative charge S for $p < p_c$). At the critical point $p = p_c$, the cluster size distribution shows the dynamic scaling $n_S(t) = S^{-5/3} f(S/t^{3/4})$ with different exponents from the Scheidegger model. The correlation time t_c scales as $t_c \approx \Delta p^{-\nu}$ ($\nu = 1.21 \pm 0.02$) where $\Delta p = |p - p_c|$.

Recently, there has been increasing interest in scaling structures of growth processes such as cluster–cluster aggregations (CCA), diffusion-limited aggregations (DLA), rough surfaces and river networks [1–12]. Considerable work has already been performed on the statistical properties of aggregation. For many aggregating systems, it is known that the cluster size distribution follows a dynamic scaling [13–16]. For example, in the Scheidegger model which is one of the simplest aggregation models [10, 17], the cluster mass distribution satisfies the dynamic scaling law [18]

$$n_S(t) \approx S^{-\tau} f(S/t^z) \qquad \text{with} \quad \tau = \tfrac{4}{3} \quad \text{and} \quad z = \tfrac{3}{2} \tag{1}$$

where the dynamic exponent z is given by the exponent of the drainage basin area in the Scheidegger river model. The exponent $\tau = \frac{4}{3}$ has been proved to be rigorous [17, 19]. Also, the exponents satisfy the scaling relationship [18]

$$(2 - \tau) z = 1. \tag{2}$$

Very recently, Takayasu [20] presented a kinetic aggregation model in which the basic dynamical variable is a 'charge' which can assume both positive and negative values. Intuitively, one can interpret each cluster having a positive or negative 'charge' which is conserved when two particles collide. It was theoretically shown that the distribution of the charge S satisfies the power law for random positive and negative injection

$$n_S \approx S^{-4/3} \qquad \text{for } \langle I \rangle > 0 \quad (\text{or } \langle I \rangle < 0) \tag{3}$$

$$n_S \approx S^{-5/3} \qquad \text{for } \langle I \rangle = 0 \tag{4}$$

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where $\langle I \rangle$ is the mean value of the injected charges, n_S is the distribution of clusters with positive charges for $\langle I \rangle > 0$, n_S is the distribution of clusters with negative charges for $\langle I \rangle < 0$ and for $\langle I \rangle = 0$ equation (4) is satisfied for both positive and negative clusters.

We consider the aggregation process with varying continuously the fraction p of random injection of positive and negative charges. For $p > \frac{1}{2}$, the clusters with positive charges dominate, while for $p < \frac{1}{2}$ the clusters with negative charges dominate. One can expect that a kinetic growth transition occurs at the critical concentration $p_c = \frac{1}{2}$. However, the kinetic growth transition has not been found until now. It is an open question as to whether or not the kinetic growth transition occurs. The charge distribution n_S has been analytically proved to satisfy (3) and (4). However, it is also an open question as to whether the charge distribution n_S satisfies the dynamic scaling. At $p_c = \frac{1}{2}$, the dynamic exponent z has also been unknown until now.

In this letter, we explore the simple aggregation model of charged particles introduced by Takayasu [20] by using computer simulation. We study the time evolution of the mean cluster size (characterized by the charge) and the cluster size distribution by varying the injection fraction p. We show that the kinetic growth transition between the cluster growth of positive charges and that of negative charges occurs at the critical fraction $p_c = \frac{1}{2}$. Also, we derive the scaling exponent ν of the correlation time t_c which scales at $t_c \approx \Delta p^{-\nu}$. We show that the charge distribution n_S obeys the dynamic scaling form $n_S(t) = S^{-5/3} f(S/t^{3/4})$ at $p_c = \frac{1}{2}$.

First we describe the simple aggregation model of charged particles introduced by Takayasu [20]. We consider the aggregation process in a discretized space and time. On every site there is at most one particle. If two particles happen to hop onto one site, they immediately coalesce into a single particle with the charge of the product equal to the sum of the charges of the two incident particles. Let $S(j, t)$ be the charge of the particle on the site j at the t time step. The aggregation can be represented by the stochastic equation for $S(j, t)$

$$S(j, t+1) = w_j(t) S(j, t) + [1 - w_{j+1}(t)] S(j+1, t) + I(j, t) \tag{5}$$

where $I(j, t)$ indicates the charge injected at the jth site at time t, $w_j(t)$ is a stochastic variable which is equal to 1 with probability $\frac{1}{2}$ when the particle on the jth site jumps to the jth site and which is equal to 0 with probability $\frac{1}{2}$ when the particle on the jth site jumps to the $(j+1)$th site. A positive charge $+1$ or a negative charge -1 are injected randomly into each site per unit time. The stochastic variable $I(j, t)$ is equal to $+1$ with probability p and is equal to -1 with probability $1-p$.

In the aggregation process governed by (5), each site has the three states: a positive charge, a negative charge and no charge (empty site). Each cluster is infinitesimal but has a finite charge. The cluster is defined by the quantity of charge. Always, a unit $(+1)$ of positive charges or a unit (-1) of negative charges are injected into every site at each time. Even on a site without a particle, a positive charge $+1$ is injected with probability p and a negative charge -1 is injected with probability $1-p$. The number of clusters increases with time since the charges are injected at a constant rate. We perform the computer simulation of (5) for the one-dimensional lattice with 10 000 sites until the time steps 20 000. The charge $S(j, t+1)$ on each site is calculated under a periodic lateral boundary condition. We calculate the mean cluster size $\langle S(t) \rangle$ which is defined as

$$\langle S(t) \rangle = \sum_S n_S S^2 \Big/ \sum_S n_S S \tag{6}$$

where n_S is the cluster size distribution characterized by the charge of the cluster. The cluster size distribution n_S can be independently defined for the cluster with positive charges and the cluster with negative charges. Figure 1 shows the log–log plot of the mean cluster size with positive charges against the time t for various p. For $p > \frac{1}{2}$, the mean cluster size $\langle S \rangle$ approaches to the straight line with the slope of the value 1.48 ± 0.02. The value 1.48 ± 0.02 of the slope agrees with the dynamic exponent z in the Scheidegger river model. At $p = \frac{1}{2}$, the mean cluster size becomes the straight line with the slope 0.75 ± 0.02. For $p < \frac{1}{2}$, the mean cluster size approaches a constant value. At $p = p_c = \frac{1}{2}$, we find the kinetic growth transition from the cluster growth of negative charges to that of positive charges with increasing p. When the injection fraction p is larger than the critical value $p_c = \frac{1}{2}$, the clusters with positive charges become dominant and the negative clusters stop growing while, if p is smaller than $p_c = \frac{1}{2}$, the clusters with negative charges dominate and the positive clusters stop growing. The kinetic growth transition is due to the competition between the positive and negative charges. We define the crossover time t_c as the point at which the tangential line of the slope 0.75 intersects with that of the slope 1.48. We can determine the crossover time t_c by finding the intersecting point in figure 1. Figure 2 shows the log–log plots of the

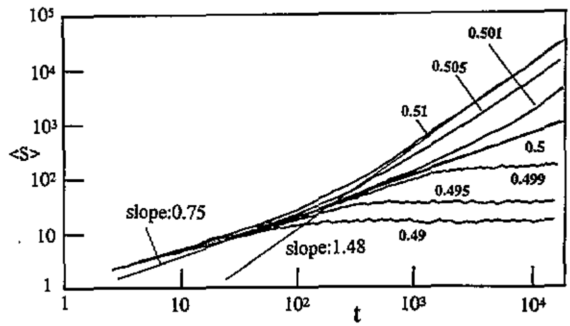

Figure 1. The log–log plot of the mean size $\langle S \rangle$ of the cluster with positive charges against the time t for various injection fraction p. For $p > \frac{1}{2}$, the mean cluster size $\langle S \rangle$ approaches to the straight line with the slope 1.48 ± 0.02. At $p = \frac{1}{2}$, the mean cluster size becomes the straight line with the slope 0.75 ± 0.02.

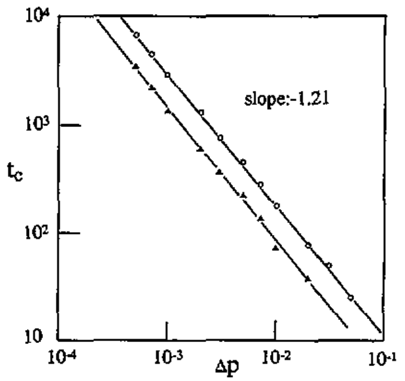

Figure 2. The log–log plot of the crossover time (correlation time) t_c against Δp. The crossover time is indicated by the circles for $p > p_c$ and by the triangles for $p < p_c$. The data points are on the straight lines with the slope -1.21 ± 0.02.

crossover time t_c against Δp ($\Delta p = p - p_c$ for $p > p_c$ and $\Delta p = p_c - p$ for $p < p_c$). The crossover time t_c for $p > p_c$ is indicated by the circles and t_c for $p < p_c$ is represented by the triangles. For both $p > p_c$ and $p < p_c$, the crossover time t_c scales as

$$t_c \approx (\Delta p)^{-\nu} \qquad \text{with } \nu = 1.21 \pm 0.02. \tag{7}$$

The crossover time t_c gives a characteristic time and represents the correlation time. At longer time scales than the correlation time t_c, the clusters dominated by positive charges appear for $p > p_c$ and the clusters with negative charges appear dominantly for $p < p_c$. At shorter time scales than the correlation time t_c, the mixed phase with both positive and negative clusters exists. For $p > p_c$, the mean cluster size $\langle S(t) \rangle$ of positive charges scales as

$$\langle S(t) \rangle \approx \begin{cases} t^{0.75} & \text{for } t < t_c \\ t^{1.48} & \text{for } t > t_c. \end{cases} \tag{8}$$

At $p = p_c$, $\langle S(t) \rangle$ scales as

$$\langle S(t) \rangle \approx t^{0.75}. \tag{9}$$

For $p < p_c$, the mean cluster size $\langle S(t) \rangle$ scales as

$$\langle S(t) \rangle \approx \begin{cases} t^{0.75} & \text{for } t < t_c \\ \text{constant} & \text{for } t > t_c. \end{cases} \tag{10}$$

We study the scaling behaviour of the cumulative cluster size distribution. The cumulative cluster-size distribution N_S is defined as

$$N_S \equiv \sum_{S'=S}^{\infty} n_{S'} \tag{11}$$

where n_S is the cluster size distribution of positive charges or negative charges. Figure 3 shows the log–log plot of the cumulative size distributions N_S against size $|S|$ for $p = 0.5$ and $p = 0.51$ at $t = 2000$. The size distributions of positive and negative charges are indicated respectively by the black and white circles. For $p = 0.5$, both positive and negative charge distributions are consistent with each other and are on the straight line with the slope $-\frac{2}{3}$ for $|S| < 100$. For $p = 0.51$, the positive charge distribution deviates from the negative charge distribution. The positive charge distribution is on the straight line with the slope $-\frac{1}{3}$ for $S < 1000$ which is consistent with that of the Scheidegger model. For $p > p_c$, the positive charge distribution crosses from the size

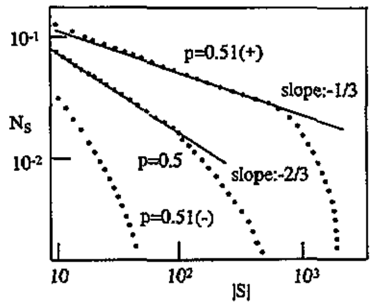

Figure 3. The log–log plot of the cumulative cluster size distributions N_S of positive and negative charges against the size $|S|$ at $t = 2000$ for $p = 0.5$ and $p = 0.51$.

distribution of the coexisting cluster structure (the straight line of the slope $-\frac{2}{3}$) to that of the Scheidegger model (the straight line of the slope $-\frac{1}{3}$) with increasing time t. In order to investigate the dynamic scaling behaviour of the charge distribution at $p=p_c$, we calculate the cumulative positive charge distribution with increasing time t. Figure 4 shows the cumulative positive charge distribution N_S against charge S for $t=2000$, 4000, 8000 and 16 000 at $p=p_c=\frac{1}{2}$. With increasing time, the cumulative distribution comes to exist on the straight line of the slope $-\frac{2}{3}$. The mean cluster size $\langle S\rangle$ scales as (9). The cumulative cluster size distribution scales as $N_S \approx S^{-2/3}$. We plot the rescaled cumulative size distribution against the rescaled size. Figure 5 shows the log–log plot of the rescaled cumulative size distribution $S^{2/3}N_S$ against the rescaled size $t^{-3/4}S$ for various t. All data points collapse on a single curve. At $p=p_c=\frac{1}{2}$, the cluster size distribution n_S satisfies the dynamic scaling

$$n_S(t)=S^{-5/3}f(S/t^{3/4}). \tag{12}$$

Here, the scaling relation between τ and z holds

$$(2-\tau)z=\tfrac{1}{4}. \tag{13}$$

The scaling relationship is compared with $(2-\tau)z=1$ of the Scheidegger model [18]. The scaling structure at $p=p_c$ is different from the Scheidegger model and the Takayasu model belongs to a different universality class from the Scheidegger model. For $p>p_c$,

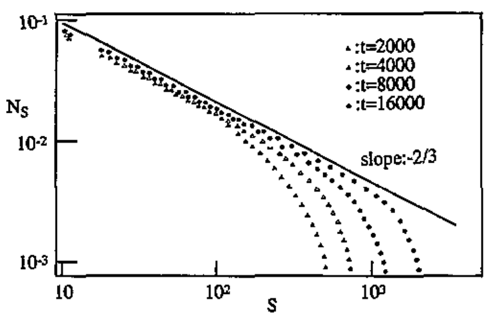

Figure 4. The time evolution of the cumulative size distribution N_S of positive charges at the critical point $p=p_c=\frac{1}{2}$. The log–log plot of the cumulative size distribution against the size S at $t=2000$, 4000, 8000 and 16 000.

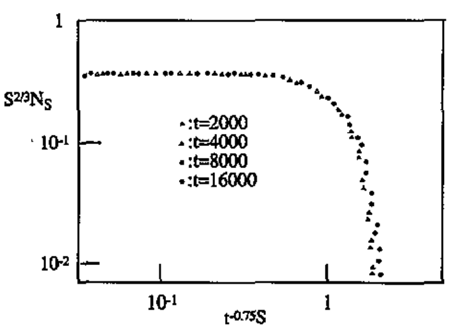

Figure 5. The log–log plot of the rescaled cumulative size distribution $S^{2/3}N_S$ against the rescaled time $t^{-0.75}S$ at $p=p_c=\frac{1}{2}$.

the cluster size distribution of positive charges shows the same dynamic scaling as the Scheidegger model [18] at longer timescales than the correlation time t_c

$$n_S(t) = S^{-4/3} f(S/t^{3/2}). \tag{14}$$

Takayasu [20] exactly solved the aggregation of charged particles and found only the exponent τ of the charge distribution (equations (3) and (4)). The kinetic growth transition found in this letter has been unknown until now. It may be possible to solve exactly this kinetic growth transition.

In summary, we find a kinetic growth transition between the cluster growth of positive charges and that of negative charges with varying injection fraction p of charged particles in the Takayasu model. We show that the charge distribution satisfies the dynamic scaling (12) at the critical point $p = p_c = \frac{1}{2}$. We derive that the correlation time t_c scales as $t_c \approx \Delta p^{-1.21}$.

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